Additional Notes on Transfer in Turbulent Pipe Flow

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A prior note by the author (1972) showed that wall region mass transfer for turbulent pipe flow is represented by the equation

$$k^+_{EW} = 0.065\ N_{Sc}^{-2/3} \tag{1}$$

An earlier paper (1971) showed that the form of Equation (1) is consistent with a developing boundary layer model derived from the equation

$$\frac{kx}{D} = 0.332 \left(\frac{x\ U_\infty}{\nu} \right)^{1/2} (N_{Sc})^{1/3} \tag{2}$$

Combination of Equations (1) and (2) with substitution of $U_\infty = 13u^*$ to correspond to $y^+ = 35$, yields

$$\frac{u^* x}{\nu} = 338 \tag{3}$$

Shaw and Hanratty (1964) obtained frequency data for the wall region which indicate disturbance velocities equivalent to the friction velocity. This corresponds to a laminar layer of $y^+ = 2.0$ with an average velocity in this laminar layer equal to the shear velocity. Popovich and Hummel (1967) report a study of the wall region which indicates a layer $y^+ = 1.6 \pm 0.4$ in which there is always a linear velocity gradient, but the slope of the gradient changes with time. Assumption of the shear velocity as the average velocity for this laminar sublayer with Equation (3) gives

$$\frac{u^{*2} t}{\nu} = 338 \tag{4}$$

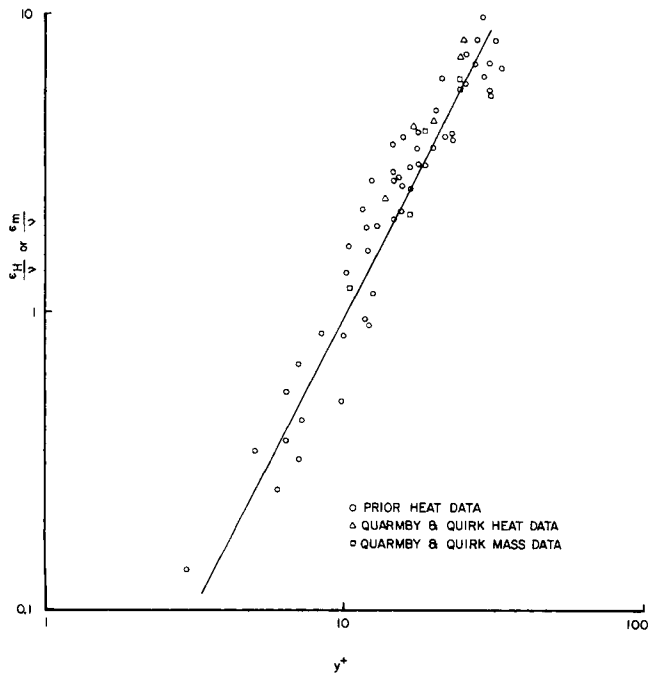

Fig. 1. Eddy diffusivity for the transition region.

Meek (1972) analyzed data for the mean period of fluctuations near the wall and concluded that the best value of the dimensionless sublayer period for fully developed turbulent flow is essentially that represented by Equation (4).

The penetration theory could be expected to apply to the transition region. Substitution of the contact time from Equation (4) in the Equation $k = 2\sqrt{D/\pi t}$ yields

$$k^+_{ET} = 0.0615\ N_{Sc}^{-1/2} \tag{5}$$

which corresponds to the eddy diffusivity

$$\frac{\epsilon}{\nu} = 0.0094\ y^{+2} \tag{6}$$

Figure 1 shows eddy diffusivity for heat data reported in the 1971 paper with the recent heat and mass data of Quarmby and Quirk (1972). Equation (6) is observed to be a good representation of the heat and mass data.

Equation (1) corresponds to the eddy diffusivity equation

$$\frac{\epsilon}{\nu} = 0.0049\ y^{+3} \tag{7}$$

Equations (7) and (6) can be assumed to represent eddy diffusivity for momentum for the regions $0 < y^+ < 2$ and $2 < y^+ < 35$, respectively, to obtain the dimensionless velocity profile. Figure 2 shows this calculated profile with experimental velocity data reported in the 1971 paper. Excellent agreement is observed to y^+ of about 12 with apparent deviation between $y^+ = 12$ and $y^+ = 35$. Quarmby and Quirk concluded that heat and mass diffusivities were equal but that the ratio of heat and mass to momentum diffusivities have a value of about two near the wall but varies smoothly to unity at the center of the pipe. Figure 2 also shows the velocity profile for $\epsilon_M = \epsilon_H/2$. Comparison with experimental velocity data shows that the equivalent diffusivities better fit the data in the wall region.

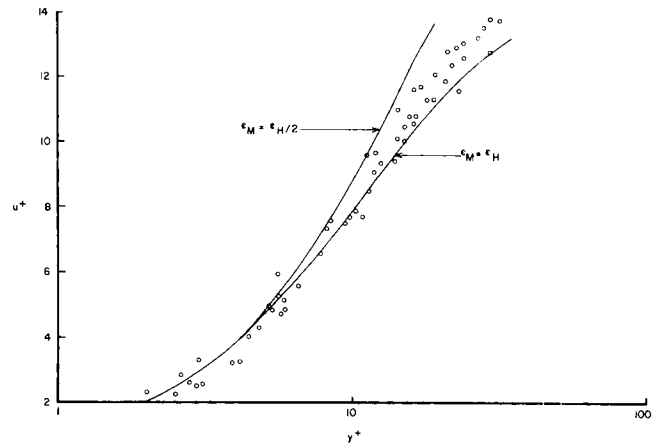

Fig. 2. Velocity profile.

NOTATION

D = molecular diffusivity
k = mass transfer coefficient
k^+ = dimensionless mass transfer coefficient, k/u^*
N_{Sc} = Schmidt number
t = eddy contact time
U_∞ = free stream velocity
u = velocity in axial direction
u^+ = u/u^*
u^* = shear velocity
x = length in axial direction
y = radial distance from wall
y^+ = yu^*/ν

Greek Letters

ϵ = eddy diffusivity
ν = kinematic viscosity

Subscripts

ET = eddy, transition region
EW = eddy, wall region
H = heat
m = mass
M = momentum

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Multicomponent Rates of Sorption of SO_2 and CO_2 in Sodium Mordenite

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Recently diffusion in binary sorbate systems has been considered. However, in almost all the cases considered, either one of the components was assumed to be inert or counter diffusion was assumed (Roberts and York, 1967; Satterfield and Katzer, 1970, 1971; Kondis and Dranoff, 1970, 1971).

Kokoszka (1970) studied the rate of sorption of propane and butane mixtures from helium by 5A molecular sieves. He reported that the rates of sorption of a tertiary system of propane and butane in helium were lower than the respective rates of sorption as pure components. Riekert (1971) studied the rate of exchange between CO_2 and C_2H_6 in hydrogen and sodium mordenites. He found that a countercurrent migration was possible in both zeolites.

Habgood (1958) studied the diffusion of mixtures of nitrogen and methane in 4A molecular sieves. He observed that nitrogen diffused faster than methane and was preferentially sorbed at the beginning. However, it was later displaced by methane, resulting in the observation of a maximum in the amount of nitrogen sorbed. Based on the chemical potential as the driving force for diffusion, Round et al. (1966) presented a numerical solution to the equations describing the sorption of a binary mixture.

EXPERIMENTAL APPARATUS AND PROCEDURE

Sorption rates were measured in a well-stirred constant volume sorption chamber consisting of a cylindrical aluminum vessel of 3-liter capacity. The agitator shaft with a steel bar attached to its bottom end was magnetically driven by a one-half horsepower D.C. motor at a controlled and measured speed. The impeller was constructed of stainless steel with two equally-spaced adjustable baskets around the shaft in which the sorbent pellets could be placed, a design similar to that used by Carberry (1961) and Ma (1967). An aluminum propeller was also attached to the shaft to ensure adequate mixing. Three equally-spaced half-inch stainless steel rods were employed as baffles with clearance between the baffles and the chamber wall. Details of the apparatus were described by Kokoszka (1970) and Roux (1972). A schematic diagram of the sorption system is shown in Figure 1.

The pellets were placed in the baskets and regenerated under a vacuum of about 25 μm Hg and at a temperature of